Journal of Pure and Applied Algebra 28 (1983) 211-212
North-Holland Publishing Company

AN ADDENDUM TO "QUADRATIC FORMS OVER POLYNOMIAL EXTENSIONS OF RINGS OF DIMENSION ONE"

Journal of Pure and Applied Algebra 24 (1983) 293–302

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Communicated by H. Bass
Received 8 November 1982

In Theorem 3.3 the condition that disc q is extended from R is redundant in view of the following:

Proposition. *Let R be a reduced commutative Noetherian ring of dimension one in which 2 is invertible and which has finite normalisation. Then the inclusion $R \hookrightarrow R[X_1, \dots, X_n]$ induces an isomorphism* $\operatorname{Disc} R \xrightarrow{\sim} \operatorname{Disc} R[X_1, \dots, X_n]$.

To prove the proposition we need the following:

Lemma. *Let R be any commutative ring in which 2 is a non-zero divisor; then $\mu_2(R) = \mu_2(R[X])$.*

Proof. Let $f = a_0 + a_1X + \cdots + a_rX^r \in \mu_2(R[X])$. Then the equation

$$(a_0 + a_1X + \cdots + a_rX^r)^2 = 1$$

gives $a_0^2 = 1$ and hence a_0 is in $\mu_2(R)$. Let, if possible, $i > 0$ be the least integer such that $a_i \neq 0$. Then $2a_0a_i = 0$ which implies that $a_i = 0$, a contradiction. Thus $f = a_0$.

Proof of the proposition. Let $\bar{R}$ be the integral closure of R in its total quotient ring and $\mathfrak{c}$ be the conductor of R in $\bar{R}$. We then have the following commutative diagram of exact sequences:

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$$\begin{tikzcd}
\mu_2(\bar{R})\oplus\mu_2(R/_C) \arrow[r] \arrow[d, "i_1"] & \mu_2(\bar{R}/_C) \arrow[r] \arrow[d, "i_2"] & \operatorname{Disc} R \arrow[r] \arrow[d, "i_3"] & \operatorname{Disc}\bar{R}\oplus\operatorname{Disc} R/_C \arrow[r] \arrow[d, "i_4"] & \operatorname{Disc}\bar{R}/_C \arrow[d, "i_5"] \\
\mu_2(\bar{R}[X])\oplus\mu_2(R/_C[X]) \arrow[r] & \mu_2(R/_C[X]) \arrow[r] & \operatorname{Disc} R[X] \arrow[r] & \operatorname{Disc}\bar{R}[X]\oplus\operatorname{Disc} R/_C[X] \arrow[r] & \operatorname{Disc}\bar{R}/_C[X]
\end{tikzcd}$$

where the vertical maps are induced by inclusions and X denotes the tuple $(X_1,\dots,X_n)$. Since $\bar{R}$ is a product of Dedekind domains and $\dim R/_C=\dim \bar{R}/_C=0$, i_1, i_2, i_4 and i_5 are isomorphisms. Thus, by the five-lemma i_3 is an isomorphism.

In view of this, Corollary 3.5 should read as $W(R[X])\xrightarrow{\sim} W(R)$.